

Model-independent Analyses of Dark-Matter Particle Interactions

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Abstract

A model-independent treatment of dark-matter particle elastic scattering has been developed, yielding the most general interaction for WIMP-nucleon low-energy scattering, and the resulting amplitude has been embedded into the nucleus, taking into account the selection rules imposed by parity and time-reversal. One finds that, in contrast to the usual spin-independent/spin-dependent (SI/SD) formulation, the resulting cross section contains six independent nuclear response functions, three of which are associated with possible velocity-dependent interactions. We find that current experiments are four orders of magnitude more sensitive to derivative couplings than is apparent in the standard SI/SD treatment, which necessarily associated such interactions with cross sections proportional to $v_T^2 \sim 10^{-6}$, where v_T is the WIMP velocity relative to the center of mass of the nuclear target.

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Keywords:

1. Introduction

The experimental community is engaged in an effort to characterize the properties of dark matter using nuclear recoil as the signal for the elastic scattering of heavy dark matter particles off nuclear targets. While astrophysics has established certain basic properties of dark matter – that it is long-lived or stable, cold or warm, gravitationally active, but without strong couplings to itself or to baryons – the nature of the interactions of dark-matter particles with ordinary matter is otherwise unknown [1, 2]. Among the leading dark-matter candidates are weakly interacting massive particles (WIMPs): it was noted long ago that such weak-scale particles could be produced in the Big Bang with abundances on the order of that required by observation, $\Omega_{DM} \sim 0.27$. Additional motivation for WIMPs comes from expectations that new physics at the mass-generation scale of the standard model might naturally lead to new particles of mass ~ 10 GeV - 10 TeV.

One of the promising methods for detecting WIMPs is via their elastic scattering with heavy nuclei (“direct detection”) [3, 4, 5]. The WIMPs would be slowly moving today, with velocities $v_T \sim 10^{-3}$, and consequently with momenta $q \sim 100$ MeV/c and kinetic energies $\epsilon \sim 50$ keV. The momentum scale is on the order of the inverse size of the nucleus, $\hbar/R_{\text{nucleus}} \sim \hbar/(1.2 fm A^{1/3}) \sim 30$ MeV/c, while ϵ is small compared to the typical excitation energies in a nucleus of ~ 1 MeV. Consequently

- the detection of WIMP elastic interactions with a nucleus requires sensitivity to recoil energies on the order of 10s of keV;

- inelastic interactions need only be considered in unusual cases where a target nucleus has an excited state within ~ 100 keV of the ground state; and
- a proper quantum mechanical treatment of the elastic scattering cross section should take into account the size of the nucleus, as $qR_{\text{nucleus}} \gtrsim 1$.

Because the WIMP will, in most cases, only scatter elastically, one also sees that parity and time-reversal selection rules that operate for diagonal matrix elements will limit what can be learned in direct detection experiments.

While we know little about dark matter interactions with ordinary matter, their possible associations with electroweak interactions suggests using the standard model as a guide. In electromagnetism, elastic scattering can occur through charge or magnetic interactions. Both interactions involve nontrivial isospin – the charge coupling is only to protons, while the magnetic coupling involves the distinct proton and neutron magnetic moments. Magnetic elastic scattering occurs through two interfering three-vector operators, spin $\vec{\sigma}(i)$ and orbital angular momentum $\vec{\ell}(i)$. For weak interactions, the weak charge operator couples primarily to neutrons, while the axial-charge operator $\vec{\sigma}(i) \cdot \vec{p}(i)$ makes effectively no contribution to elastic scattering, apart from small recoil corrections, due to the constraints imposed by parity and time-reversal invariance. One might expect, consequently, that the WIMP-nuclear interaction will involve a variety of operators as well as couplings that depend on isospin.

In part for historical reasons, WIMP elastic scattering experiments are most often analyzed by assuming the interaction is simpler than those described above: isoscalar, coupled either to the nucleon number operator $1(i)$ (spin-independent or SI) or the nucleon spin $\sigma(i)$ (spin-dependent or SD) [3, 6, 7]. These are the operators for a point nucleus. While a form factor is often introduced to account phenomenologically for the fact that the momentum transfer is large on the nuclear scale, the quantum mechanical consequences of $o(1)$ operators like $\vec{q} \cdot \vec{p}(i)$ have been largely neglected.

Recently there have been efforts to treat the WIMP-nucleon interaction in more generality, using the tools of effective field theory (EFT) [8, 9, 10, 11]. We describe the approach of [9, 11] in Sec. 2 and its consequences for WIMP-nucleus elastic scattering. Consistent with general symmetry arguments, six independent nuclear response functions are identified, in contrast to the two assumed in SI/SD treatments. The new operators are associated with derivative couplings, where a proper treatment of $\vec{q} \cdot \vec{p}(i)$ is essential due to the need to identify associated parity- and time-reversal-conserving elastic operators. When this is done, we find that velocity-dependent interactions lead to cross sections $\sim q^2/m_N^2 G_F^2 \sim 10^{-2} G_F^2$, where m_N is the nucleon mass and G_F the weak coupling constant, in contrast to the SI/SD result, $\sim v_T^2 G_F^2 \sim 10^{-6} G_F^2$. Our effective theory treatment shows that much more can be learned about the properties of WIMP dark matter from elastic scattering experiments than is generally appreciated. However, it also shows that a greater variety of experiments will be necessary to extract this information and to eliminate possible sources of confusion, when competing experiments are compared.

2. The Nuclear Elastic Response from Effective Theory

Here we summarize the effective theory construction of the WIMP-nucleon interaction of Ref. [9, 11]. Details can be found in the original papers. The Lagrangian density for the scattering of a WIMP off a nucleon is taken to have the form

$$\mathcal{L}_{\text{int}}(\vec{x}) = c \Psi_{\chi}^*(\vec{x}) O_{\chi} \Psi_{\chi}(\vec{x}) \Psi_N^*(\vec{x}) O_N \Psi_N(\vec{x}), \quad (1)$$

where the $\Psi(\vec{x})$ are nonrelativistic fields and where the WIMP and nucleon operators O_{χ} and O_N may have vector indices. The operators O_{χ} and O_N are then allowed to take on their most general form, constrained by imposing relevant symmetries. The construction was done in the nonrelativistic limit to second order in the momenta. Thus the relevant operators are those appropriate for use with Pauli spinors. The Galilean-invariant amplitudes take the form

$$\sum_{i=1}^N (c_i^n O_i^n + c_i^p O_i^p), \quad (2)$$

where the coupling coefficients c_i may be different for proton and neutrons. The number N of such operators O_i – which have the product form $O_{\chi}^i \otimes O_N^i$ – depends on the generality of the particle physics description.

The WIMP and nucleon operators for this construction include 1_χ and 1_N , the three-vectors \vec{S}_χ and \vec{S}_N , and the momenta of the WIMP and nucleon. Of the four momenta involved in the scattering (two incoming and two outgoing), only two combinations are physically relevant due to inertial frame-independence and momentum conservation. It is convenient to work with the frame-invariant quantities, the momentum transfer \vec{q} and the WIMP-nucleon relative velocity,

$$\vec{v} \equiv \vec{v}_{\chi,\text{in}} - \vec{v}_{N,\text{in}}. \quad (3)$$

The Hermitian form of this velocity is

$$\vec{v}^\perp = \vec{v} + \frac{\vec{q}}{2\mu_N} = \frac{1}{2} (\vec{v}_{\chi,\text{in}} + \vec{v}_{\chi,\text{out}} - \vec{v}_{N,\text{in}} - \vec{v}_{N,\text{out}}) = \frac{1}{2} \left(\frac{\vec{p}}{m_\chi} + \frac{\vec{p}'}{m_\chi} - \frac{\vec{k}}{m_N} - \frac{\vec{k}'}{m_N} \right) \quad (4)$$

which satisfies $\vec{v}^\perp \cdot \vec{q} = 0$ as a consequence of energy conservation. Here μ_N is the WIMP-nucleon reduced mass, \vec{p} (\vec{p}') is the incoming (outgoing) WIMP three-momentum, and \vec{k} (\vec{k}') the incoming (outgoing) nucleon three-momentum. It was shown in [9] that operators are guaranteed to be Hermitian if they are built out of the following four three-vectors,

$$\vec{S}_\chi \quad \vec{S}_N \quad \vec{v}^\perp \quad i \frac{\vec{q}}{m_N} \quad (5)$$

The nucleon mass m_N has been introduced as a convenient scale to render \vec{q}/m_N and the constructed O_i dimensionless: the choice of this scale is not arbitrary, but is instead connected with the velocity operator when that operator is embedded in a nucleus, as we discuss later. One finds to second order in velocities and momenta,

$$\begin{aligned} O_1 &= 1_\chi 1_N \\ O_2 &= (\vec{v}^\perp)^2 \\ O_3 &= i \vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp \right) \\ O_4 &= \vec{S}_\chi \cdot \vec{S}_N \\ O_5 &= i \vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp \right) \\ O_6 &= (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}) (\vec{S}_N \cdot \frac{\vec{q}}{m_N}) \\ O_7 &= \vec{S}_N \cdot \vec{v}^\perp \\ O_8 &= \vec{S}_\chi \cdot \vec{v}^\perp \\ O_9 &= i \vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N}) \\ O_{10} &= i \vec{S}_N \cdot \frac{\vec{q}}{m_N} \\ O_{11} &= i \vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \\ O_{12} &= \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp) \\ O_{13} &= i (\vec{S}_\chi \cdot \vec{v}^\perp) (\vec{S}_N \cdot \frac{\vec{q}}{m_N}) \\ O_{14} &= i (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}) (\vec{S}_N \cdot \vec{v}^\perp) \\ O_{15} &= -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}) ((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}) \end{aligned} \quad (6)$$

The first eleven of these operators can be generated by interactions involving only spin-0 or spin-1 mediators, and were discussed in [9]. One of these, O_2 , was later eliminated from consideration in [11], as it cannot be obtained from the

leading-order non-relativistic reduction of a manifestly relativistic operator. The remaining operators are associated with more complicated exchanges. These operators can be labeled as LO, NLO, and N²LO, according to the total number of momenta and velocities they contain, corresponding to total cross sections that scale as v_T^0 , v_T^2 , or v_T^4 , where v_T is the WIMP velocity in the laboratory frame (though, below, we stress that this scaling is accompanied by additional factors that arise from the treatment of velocity-dependent interactions). Operator O_{15} is cubic in velocities and momenta, generating a total cross section of order v_T^6 (N³LO). It is retained because it arises as the leading-order nonrelativistic limit of a covariant four-fermion interaction, as discussed in [11]. Each operator can have distinct couplings to protons and neutrons. Introducing isospin, which is also useful as an approximate symmetry of the nuclear wave functions, the general form of the effective theory interaction becomes

$$\sum_{i=1}^{15} (c_i^0 1 + c_i^1 \tau_3) O_i = \sum_{\tau=0,1} \sum_{i=1}^{15} c_i^\tau O_i t^\tau, \quad c_2^\tau \equiv 0,$$

where $c_i^0 = \frac{1}{2}(c_i^p + c_i^n)$, $c_i^1 = \frac{1}{2}(c_i^p - c_i^n)$, $t^0 = 1$, and $t^1 = \tau_3$. Thus the EFT has a total of 28 parameters, associated with 14 space/spin operators each of which can have distinct couplings to protons and neutrons.

This interaction can then be embedded in the nucleus. The procedure followed in [9, 11] assumes that the nuclear interaction is the sum of the WIMP interactions with the individual nucleons in the nucleus. The nuclear operators then involve a convolution of the O_i , whose momenta must now be treated as local operators appropriate for bound nucleons, with the plane wave associated with the WIMP scattering, which is an angular and radial operator that can be decomposed with standard spherical harmonic methods. Because momentum transfers are typically comparable to the inverse nuclear size, it is crucial to carry through such a multipole decomposition in order to identify the nuclear responses associated with the various c_i s. The scattering probability is given by the square of the (Galilean) invariant amplitude \mathcal{M} , a product of WIMP and nuclear matrix elements, averaged over initial WIMP and nuclear magnetic quantum numbers M_χ and M_N , and summed over final magnetic quantum numbers.

The result can be organized in a way that factorizes the particle and nuclear physics

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_N + 1} \sum_{\text{spins}} |\mathcal{M}|^2 \equiv \sum_k \sum_{\tau, \tau'=0}^1 R_k \left(v_T, \frac{q^2}{m_N^2}, \{c_i^\tau c_j^{\tau'}\} \right) W_k^{\tau\tau'}(q^2 b^2) \quad (7)$$

where the sum extends over products of WIMP response functions R_k and nuclear response functions W_k . The R_k isolate the particle physics: they depend on specific combinations of bilinears in the low-energy constants (LECs) of the EFT – the $2N$ coefficients of Eq. (2) – labeled by isospin τ (isoscalar, isovector). The WIMP response functions also depend on the magnitude of the WIMP velocity v_T , measured relative to the center of mass of the nucleus, and the three-momentum transfer $\vec{q} = \vec{p}' - \vec{p} = \vec{k} - \vec{k}'$. The nuclear response functions W_k can be varied by experimentalists, if they explore a variety of nuclear targets. The W_k are functions of $y \equiv (qb/2)^2$, where b is the nuclear size (explicitly the harmonic oscillator parameter if the nuclear wave functions are expanded in that single-particle basis).

The embedding of the WIMP-nucleon interaction in the nucleus will necessarily lead to the most general form of the WIMP-nucleus interaction, provided the quantum mechanics is done properly. But it is also satisfying to derive the end result from symmetry considerations alone. Our EFT operators are constructed from the nucleon charge, spin, and velocity. These operators can be combined to form a nucleon vector charge operator $1(i)$, an axial-vector charge $\vec{\sigma}(i) \cdot \vec{v}(i)$, an axial spin current $\vec{\sigma}(i)$, a vector velocity current $\vec{v}(i)$, and a vector spin-velocity current $\vec{\sigma}(i) \times \vec{v}(i)$. Charges and currents can be combined with the plane-wave operator appearing in the scattering, $e^{i\vec{q} \cdot \vec{r}(i)}$, to produce nuclear multipoles operators \hat{O}_J carrying definite angular momentum and parity, and transforming with a definite sign under time reversal. In the case of the currents, one can take three projections – longitudinal, transverse electric, and transverse magnetic – relative to a quantization axis defined by the momentum transfer. As the nuclear ground state is nearly an eigenstate of both parity and time reversal, only those multipole operators that are even under parity and time reversal can contribute. By eliminating all operators with the wrong transformation properties (a exercise familiar from standard weak interactions theory) one obtains the surviving response functions and thus the general form of the cross section. The results, shown in Fig. 1, indicate six response functions are needed to describe WIMP-nucleus elastic scattering, not the two (SI/SD) usually assumed.

Specific forms for the WIMP and nuclear response functions appearing in Eq. (7) can be obtained from the EFT Lagrangian, as detailed in Ref. [11], under the assumption that the WIMP-nucleus interaction is the sum over the

multipolarity:	Even J	Odd J
vector charge	C_J	C_J
axial charge	C_J^5	C_J^5

multipolarity:	Even J	Odd J	Even J	Odd J	Even J	Odd J
axial spin	L_J^5	L_J^5	T_{JM}^{5el}	T_{JM}^{5el}	T_{JM}^{5mag}	T_{JM}^{5mag}
vector velocity	L_J	L_J	T_{JM}^{el}	T_{JM}^{el}	T_{JM}^{mag}	T_{JM}^{mag}
vector spin-velocity	L_J	L_J	T_{JM}^{el}	T_{JM}^{el}	T_{JM}^{mag}	T_{JM}^{mag}

Figure 1. Constraints imposed by the parity and time-reversal properties of the nuclear ground state on the multipole operators mediating WIMP-nucleus elastic scattering. Transitions forbidden by parity (time reversal) are tinted red (blue); those forbidden by both are indicated by the red-blue gradient fill. Six types of multipole operators are allowed, corresponding to the even-angular-momentum- J multipoles of the vector charge operator (which generate the SI response), the odd- J longitudinal and transverse electric multipoles of the axial spin operator (which generate in combination the standard SD response), the odd- J transverse magnetic multipoles of the vector velocity operator, and the even- J longitudinal and transverse electric multipoles of the vector spin-velocity operator.

various EFT WIMP-nucleon interactions. As the arguments just given above indicate there must be six types of nuclear multipole operators in a model-independent formulation, and as our effective theory Lagrangian gives the most general WIMP-nucleon interaction, one might anticipate that a straight-forward application of that Lagrangian would then generate the explicit forms of the nuclear multipole operators. While this takes some algebra, indeed this is what happens. The nuclear response functions are generated by six operators that we can write, for simplicity, in their long-wavelength forms (that is, without accompanying form factors). One finds the leading-order multipoles

$$\begin{aligned}
 C_{J=0M} &\rightarrow \sum_{i=1}^A 1(i) & L_{J=1M}^5 &\sim T_{J=1M}^{5el} \rightarrow \sum_{i=1}^A \sigma_{1M}(i) & T_{J=1M}^{mag} &\rightarrow \frac{q}{m_N} \sum_{i=1}^A \ell_{1M}(i) \\
 L_{J=0M} &\rightarrow \frac{q}{m_N} \sum_{i=1}^A \vec{\sigma}(i) \cdot \vec{\ell}(i) & L_{J=2M} &\rightarrow \frac{q}{m_N} \sum_{i=1}^A \left[r(i) \otimes \left(\vec{\sigma}(i) \times \frac{1}{i} \vec{\nabla} \right) \right]_{2M} & T_{J=2M}^{el} &\rightarrow \frac{q}{m_N} \sum_{i=1}^A \left[r(i) \otimes \left(\vec{\sigma}(i) \times \frac{1}{i} \vec{\nabla} \right) \right]_{2M}
 \end{aligned}$$

The first two operators are the standard SI/SD interactions, though as indicated the effective theory divides the spin response into separate longitudinal (L_{JM}^5) and transverse (T_{JM}^{5el}) components. These spin responses have distinct $R_k(v_T^2, q^2/m_N^2)$ – different bilinear functions of the c_i and thus different sensitivity to the underlying EFT operators – as well as $W_k(q^2 b^2)$ characterized by distinct form factors. In addition we find three new response functions, each proportional to q/m_N (an indication that they are connected with the finite size of the nucleus), generated in two cases by familiar scalar and vector operators, $\vec{\sigma}(i) \cdot \vec{\ell}(i)$ and $\vec{\ell}(i)$, and in the third case by a tensor operator that is clearly closely related to $\vec{\sigma}(i) \cdot \vec{\ell}(i)$. For one response function, the vector longitudinal response L_{JM} , both scalar and tensor terms survive in the long wavelength limit.

One sees that the general form of the nuclear response, even in the long wavelength limit, is more complex than is apparent from the SI/SD formulation. Two scalar ($J = 0$) operators emerge from the effective theory, and three vector ($J = 1$) operators. Two sets of operators interfere, $C_{JM} - L_{JM}$ and $T_{JM}^{5el} - T_{JM}^{mag}$, as happens in familiar electroweak processes such as neutrino scattering off nuclei. The bottom line for experimentalists is that each nuclear response function is a “knob” that can be turned by picking nuclear targets with the requisite properties. Each response function is accompanied by a WIMP tensor $R_k(v_T^2, q^2/m_N^2)$ that involves a distinct set of LECs. Thus, including the two interference terms, in principle one can obtain eight constraints on the LECs of the effective theory from elastic scattering experiments. In the future, when dark matter scattering is observed, the experimental task will have been completed when those eight constraints are determined: all information on candidate fundamental theories (the ultra-violet theories) that can be extracted from inelastic scattering will have been obtained, at that point.

Experimentalists can turn the nuclear physics “knobs” by selecting nuclear targets that isolate the various response functions, thereby determining the associated $R_k(v_T^2, q^2/m_N^2)$. For example, if one wants to extract the WIMP physics

associated with the two response functions generated by $\vec{\sigma}(i)$ and $\vec{\ell}(i)$, one could employ two odd-A nuclear targets, one having its unpaired valence nucleon in an $\ell = 0$ orbit (an s-wave orbit), the other with a valence nucleon in a high- ℓ orbit. The former would be sensitive only to $\vec{\sigma}(i)$, the latter primarily to $\vec{\ell}(i)$. In fact, one should do four such experiments two with targets having unpaired valence protons and two with targets having unpaired valence neutrons, to test the isospin of these couplings. With such a strategy, extended to the full set of nuclear response functions, one would eventually learn everything than can be learned from elastic scattering.

3. The Point-Nucleus Limit: The Origin of the Additional Response Functions

Before turning to the $R_k(v_T^2, q^2/m_N^2)$, it is interesting to explore the origin of the additional response functions. The standard SI/SD form of the cross section can be obtained (apart from the detail that one should allow the longitudinal and transverse projections of spin to have distinct coefficients) by taking the point nucleus limit. Naively one might hope that this point-nucleus SI/SD form corresponds to some lowest-order approximation, capturing the leading-order effects of candidate effective interactions. But in fact this is not the case for about half of the operators appearing in Eq. (6), those involving the WIMP-nucleon relative velocity.

As it is easiest to illustrate the point in an example, consider

$$O_8 = \sum_{i=1}^A \vec{S}_\chi(i) \cdot \vec{v}^\perp(i), \quad \vec{v}^\perp(i) \equiv \vec{v}_\chi - \vec{v}_N(i).$$

(We have made a slight simplification by not symmetrizing the velocity between initial and final states – see [9] for details.) If we take the point-nucleus limit, all nucleons are at the same point, and the interaction reduces to

$$\vec{S}_\chi(i) \cdot \vec{v}_T \sum_{i=1}^A 1(i),$$

so that the nuclear interaction is SI with a coefficient that depends on the WIMP velocity relative to the nuclear center-of-mass, $v_T \sim 10^{-3}$. Associated cross sections, which depend on the square of this amplitude, would be of order $\sim 10^{-6}$ of the weak interaction value, consequently. However it is easy to see that this point-nucleus limit is unjustified. The original interaction contains A independent internal WIMP-nucleon velocities, which we are free to rewrite by a standard Jacobi transformation as

$$\{\vec{v}^\perp(i), i = 1, \dots, A\} \rightarrow \{\vec{v}_T; \vec{v}(i), i = 1, \dots, A-1\}$$

singling out the WIMP velocity relative to the nuclear center-of-mass, leaving $A-1$ velocities $\vec{v}(i)$ corresponding to the independent internucleon velocities. For example, for the simplest case of $A=2$, there is one such internucleon velocity, $\vec{v}_1 \equiv (\vec{v}(2) - \vec{v}(1))/\sqrt{2}$. Now internucleon velocities are typically $\sim 10^{-1}$, not $\sim 10^{-3}$. Thus the point-nucleus limit effectively keeps one amplitude that is tiny (proportional to v_T), and discards $A-1$ other velocities that couple to \vec{S}_χ with precisely the same strength, that are 100 times larger.

One might be misled into thinking that one can neglect the internucleon velocities, despite their size, because of parity: they are intrinsic nuclear operators carrying odd parity. But this overlooks the fact that while the energy transfer in WIMP scattering is small, the three-momentum transfer is large, typically $qr(i) \gtrsim 1$. Thus expanding the full nuclear operator to first order in q (while quantizing along \vec{q} and taking the $J=1$ part) yields

$$e^{i\vec{q} \cdot \vec{r}(i)} \vec{v}(i) \rightarrow -\frac{1}{i} q \vec{r}(i) \times \vec{v}(i) = -\frac{1}{i} \frac{q}{m_N} \vec{r}(i) \times \vec{p}(i) = -\frac{q}{m_N} \vec{\ell}(i)$$

One obtains a familiar dimensionless parity-conserving operator, the orbital angular momentum, accompanied by a dimensionless constant $q/m_N \sim 1/10$. That is, our derivation shows that the scale of the internal relative nucleon velocities is encoded in the parameter q/m_N .

This is then very helpful in the effective theory formulation. We mentioned previously that the most general Hermitian WIMP-nucleon interaction can be constructed from the four dimensionless variables

$$\vec{S}_\chi \quad \vec{S}_N \quad \vec{v}^\perp \quad i \frac{\vec{q}}{m_N}$$

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The reason that m_N is the natural choice for the mass scale in the fourth variable is that the parameter q/m_N naturally arises from the proper treatment of relative nucleon velocities in the nucleus. As outlined below (and discussed in greater detail in [11]), when velocity-dependent interactions are treated in a manner that properly takes into account nuclear finite size, their effects are enhanced by a factor of $\sim (\mu_T/m_N)^2 \sim 10^4$, where μ_T is the reduced mass for the WIMP-nucleus scattering, relative to treatments that take the point-nucleus limit. Thus velocity-dependent interactions that appear to be very difficult to test, in the point nucleus limit, in fact make large contributions. These enhancement are connected with new response functions generated by operators like $\vec{\ell}(i)$: by exploiting these new response functions, one can determine the strengths of such velocity-dependent interactions. In addition to misrepresenting the magnitude of the cross section, the point nucleus limit also mischaracterizes the multipolarity of the scattering. This is seen in our example, $\vec{S}_\chi \cdot \vec{v}^\perp$. The associated point-nucleus operator is $1(i)$, a scalar, while the scattering is actually dominantly vector, generated by $\vec{\ell}(i)$.

4. Constraining the Effective Theory Parameters

The WIMP-nucleus differential cross section as a function of the recoil energy E_R carried off by the scattered nucleus can be written

$$\frac{d\sigma(v_T, E_R)}{dE_R} = \frac{2m_T}{\pi v_T^2} G_F^2 \left[\frac{1}{2j_\chi + 1} \frac{1}{2j_N + 1} \sum_{\text{spins}} |\mathcal{M}^{\text{Nuc}}|^2 \right] = \frac{2m_T}{\pi v_T^2} G_F^2 \frac{4\pi}{2j_N + 1} \sum_{\tau, \tau'=0}^1 \sum_k R_k^{\tau\tau'} \left(v_T^2, \frac{q^2}{m_N^2} \right) W_k^{\tau\tau'}(q^2 b^2) \quad (8)$$

where the sum extends over the various response functions, generated by the six sets of multipole operators including two interference terms. The Fermi constant has been introduced to make the weak scale explicit. The sums over the isospin indices τ, τ' appear because the $R_k^{\tau\tau'}$ depend on the LECs

$$c_i^{\tau=0} = (c_i^{(p)} + c_i^{(n)})/2 \quad c_i^{\tau=1} = (c_i^{(p)} - c_i^{(n)})/2$$

we introduced previously. Because we have normalized these to the weak scale, the LECs are dimensionless. The conventions employed here are those of Ref. [11], where more details are given.

In this formula, the left hand side will be measured (some day, hopefully), while on the right hand side, the experimentalist can vary the $W_k^{\tau\tau'}(q^2 b^2)$, the nuclear response functions. These response functions, as $q \rightarrow 0$, are generated by squaring the nuclear matrix elements of the long-wavelength operators discussed in the previous section; for general q , they also involve nuclear form factors, which in the case of the harmonic oscillator shell model, depend on $(qb)^2$, where b is the harmonic oscillator size parameter. Consequently, in an ideal world, experimentalists would vary the $W_k^{\tau\tau'}(q^2 b^2)$ by using nuclear targets with different angular momenta J , different isospins, and different shell structures, to obtain a sufficient number of constraints to map out the eight $R_k^{\tau\tau'}(v_T^2, q^2/m_N^2)$ (six response functions plus two interference terms). One would have then obtained all of the information on dark matter particle interactions that is available from elastic scattering. Note that more than eight constraints can be obtained: the measurements can be done as a function of $E_R = q^2/2m_T$, m_T the target mass, to separate the LECs contributing to a given $R_k^{\tau\tau'}(v_T^2, q^2/m_N^2)$.

It is helpful to consider examples in which we contrast the standard SI/SD treatment with a full quantum mechanical treatment, while still employing the EFT Lagrangian. For SI scattering we find

$$R_{SI}^{\tau\tau'} = c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{q^2}{m_N^2} v_T^2 c_5^\tau c_5^{\tau'} + v_T^2 c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'} \right] \quad W_{SI}^{\tau\tau'} \xrightarrow{q \rightarrow 0} \frac{1}{4\pi} \langle j_N || \sum_{i=1}^A t^\tau(i) || j_N \rangle \langle j_N || \sum_{i=1}^A t^{\tau'}(i) || j_N \rangle \quad (9)$$

where j_χ is the WIMP spin. We see that the EFT operators \mathcal{O}_5 and \mathcal{O}_8 contribute to SI scattering, but their contributions are suppressed by $\sim 10^{-8}$ and $\sim 10^{-6}$ respectively. (\mathcal{O}_8 was the velocity-dependent operator example we discussed in the previous section.) But if the nuclear finite size is properly treated, one obtains a T^{mag} response function as well

$$R_{T^{\text{mag}}}^{\tau\tau'} = \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'} \right] \quad W_{T^{\text{mag}}}^{\tau\tau'} \xrightarrow{q \rightarrow 0} \frac{1}{24\pi} \frac{q^2}{m_N^2} \langle j_N || \sum_{i=1}^A \ell(i) t^\tau(i) || j_N \rangle \langle j_N || \sum_{i=1}^A \ell(i) t^{\tau'}(i) || j_N \rangle \quad (10)$$

The two EFT operators \mathcal{O}_5 and \mathcal{O}_8 now are associated with kinematic suppression factors of 10^{-4} and 10^{-2} , respectively. Comparing the two results, we see that a proper treatment of finite size

- allows us to separately measure the effects of c_5 and c_8 , when previously they appeared in combination with c_1 and c_{11} ;
- generates a response with increased sensitivity to these two couplings; and
- leads to a dominant rank-one response requiring $j_N > 0$, in contrast to the point-nucleus result that O_5 and O_8 contribute very weakly to the scalar SI response.

Similar in the SD point-nucleus limit

$$R_{SD}^{\tau\tau'} = \frac{1}{12} \left[\frac{q^2}{m_N^2} v_T^2 c_3^\tau c_3^{\tau'} + v_T^2 c_7^\tau c_7^{\tau'} + \frac{q^2}{m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_X(j_X + 1)}{3} \left(3c_4^\tau c_4^{\tau'} + \frac{q^2}{m_N^2} (c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'} + 2c_9^\tau c_9^{\tau'}) \right) \right. \\ \left. + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + 2v_T^2 c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^2 (c_{13}^\tau c_{13}^{\tau'} + c_{14}^\tau c_{14}^{\tau'} - c_{12}^\tau c_{15}^{\tau'} - c_{15}^\tau c_{12}^{\tau'}) + \frac{q^4}{m_N^4} v_T^2 c_{15}^\tau c_{15}^{\tau'} \right] \\ W_{SD}^{\tau\tau'} \xrightarrow{q \rightarrow 0} \frac{1}{4\pi} \langle j_N \| \sum_{i=1}^A \sigma(i) \tau(i) \| j_N \rangle \langle j_N \| \sum_{i=1}^A \sigma(i) \tau'(i) \| j_N \rangle \quad (11)$$

But the velocity-suppressed operators appearing above make much stronger contributions to, and can be extracted more cleanly from, the new response functions that arise from properly treating the structure of the nucleus. For example, for scattering off a $j_N = 0$ nucleus, the longitudinal response function is

$$R_L = \frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_X(j_X + 1)}{12} \left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau \right) \left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'} \right) \\ W_L^{\tau\tau'} \xrightarrow{q \rightarrow 0} \frac{1}{36\pi} \frac{q^2}{m_N^2} \langle j_N \| \sum_{i=1}^A \vec{\sigma}(i) \cdot \vec{\ell}(i) \tau(i) \| j_N \rangle \langle j_N \| \sum_{i=1}^A \vec{\sigma}(i) \cdot \vec{\ell}(i) \tau'(i) \| j_N \rangle \quad (12)$$

Here, as in our previous example, O_3 , O_{12} , and O_{15} make substantially larger contributions to a new scalar response than they do to the SD response. In a SI/SD treatment, one would conclude that these operators appear in a complicated way in the SD response, entwined with O_4 , O_6 , O_7 , O_9 , O_{10} , O_{13} , and O_{14} and suppressed by v_T^2 , when in fact they make the leading contributions to a new type of scalar response. In principle all three couplings could be determined from the nuclear recoil distribution, as O_3 , O_{12} , and O_{15} appear in R_L with distinct leading-order behaviors in q^2/m_N^2 .

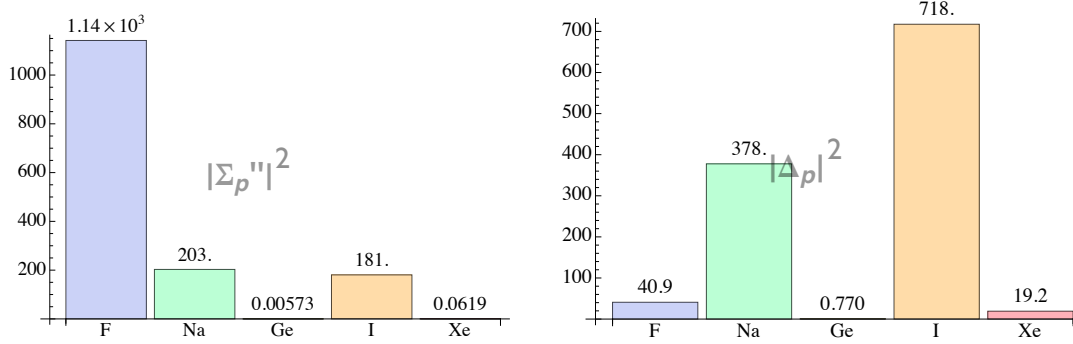
These few examples show that many more tools become available for constraining the effective theory's LECs once the new response functions associated with nuclear compositeness are recognized. Velocity-dependent interactions that appear to be difficult to probe in the standard SI/SD formalism can be isolated and studied in these new response functions, where they play enhanced roles.

5. Implications for Experiments

In the previous section we have noted that the larger number of responses that are revealed when the nuclear finite size is properly treated opens up lots of additional opportunities for constraining the parameters of the effective theory, and thus for restricting the form of candidate ultraviolet theories that map onto the EFT. Conversely, if one limits consideration to the standard SI/SD description of the scattering, one can be misled. For example, conflicts may appear to arise between experiments because the analysis framework is too restrictive, when in fact no conflict exists.

One illustration is given in Fig. 2, where the spin response function used in standard SI/SD responses is compared to the transverse magnetic (orbital angular momentum) response function, for the targets F, Na, Ge, I, and Xe. The results come from the shell-model work of [9]. Both response functions are “SD” in the sense that they involve rank-one operators – they cannot be distinguished geometrically – but as they depend on distinct underlying nuclear operators, they can be distinguished by selecting targets with the right properties. In fact there are large differences in the responses to the two underlying operators for targets currently in use in dark matter studies. For example, if the operators couple to protons, the ratio in the responses of iodine (e.g., DAMA, KIMS, COUPP) and fluorine

Vector, proton coupled: $\vec{\sigma}(i)$ vs. $\vec{\ell}(i)$



Vector, neutron coupled: $\vec{\sigma}(i)$ vs. $\vec{\ell}(i)$

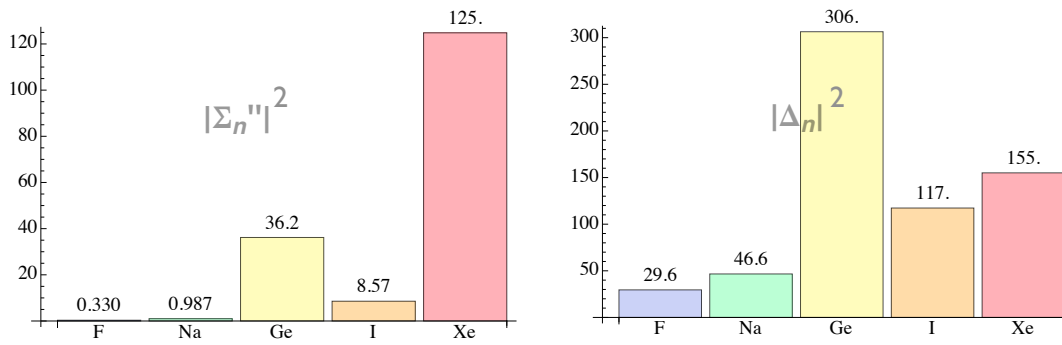


Figure 2. A comparison of the spin response function (the longitudinal and transverse electric components of an axial spin operator, left frames), employed in SI/SD analyses, with the orbital angular momentum response function (the transverse magnetic component of the vector velocity operator, right frames). The upper (lower) frames assume a coupling to protons (neutrons). The calculations are taken from the shell-model work of [9, 11].

(e.g., PICASSO, PICO, COUPP) for $\langle \vec{\sigma} \rangle^2$ vs. $\langle \vec{\ell} \rangle^2$ differ by ~ 110 . For couplings to neutrons, the responses of Xe (e.g., LUX, Xenon, XMASS) are similar for spin and orbital angular momentum, but those for I and Ge (e.g., CDMS, COGENT, Edelweiss) change by an order of magnitude, while those for Na (e.g., DAMA, ANAIS) and F change by two orders of magnitude. Consequently, if one analyzes experiments assuming a SD coupling, but in fact the underlying operator is $\vec{\ell}(i)$, experiments that are consistent may appear to be in conflict. Properly interpreted, however, one would find instead evidence that the underlying WIMP-nucleon interaction is associated with $\vec{\ell}(i)$, not $\vec{\sigma}(i)$; this ansatz could then be tested in a follow-up experiment.

The field is entering the “G2” phase of dark matter elastic scattering experiments: a smaller number of experiments may be done, but with larger detector masses. Our effective theory arguments, however, argue for continued development of detectors using a variety of nuclear targets. If dark matter is seen in direct-detection experiments, there is no doubt that a large set of experiments will be needed: as established here, a great deal of information can be extracted from such experiments because of the multiple response functions that contribute. Thus it is important, even if only a few detectors are in operation, to continue developing new technologies that can be deployed when needed. One technique and one target will not answer all of the question. Even early in the discovery phase, it will be important to analyze experiments in the kind of general framework provided here. Repeating an example from the paragraph above, if rates in a Ge detector appear to be high compared to those seen in a Xe detector, this might

suggest $\tilde{\ell}(i)$ as an operator and thus influence the choice of a third detector.

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